Supplementary material: quantitative analysis of free-decay data

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I. INTRODUCTION

Stalp *et al.* [1] were the first to recognize that the $L \propto t^{-3/2}$ dependence during free decay could be due to the saturation of the size of the energy-containing eddy by the container size. They introduced a method of interpreting the prefactor A in terms of the value of ζ for HIQT. Since then this method has also been used by several groups [2–5]. It assumes that wall-bounded turbulence can be described by a single energy spectrum E_k in the space of wavenumbers k. For $k > k_1$ (where $k_1 = 2\pi/d$ in [1]), this spectrum is taken to be equal to the Kolmogorov spectrum for HIT [6–8],

$$E_k = C\epsilon^{2/3}k^{-5/3} \tag{1}$$

(here $\epsilon = -\dot{\mathcal{E}}$ is the energy flux and $C \approx 1.5$ is the Kolmogorov constant [9]), while for $k < k_1$, $E_k = 0$, because modes of size greater than the container do not exist. The total energy per unit volume is thus

$$\mathcal{E} \approx \int_{k_1}^{\infty} E_k dk = \frac{3}{2}C\epsilon^{2/3}k_1^{-2/3}, \tag{2}$$

and its rate of change during a quasi-steady decay is

$$\dot{\mathcal{E}} = -\epsilon = -(3C)^3 k_1^{-2} t^{-3} \tag{3}$$

at late time t. Equating this with Eq. 1 from the main paper, we arrive at

$$L(t) = (3C)^{3/2}\zeta^{-1/2}k_1^{-1}(\kappa t)^{-3/2}. \tag{4}$$

In other words, this model has just one free parameter, k_1, which depends on the container size and, perhaps, boundary conditions (BC). The lifetime of the eddy is

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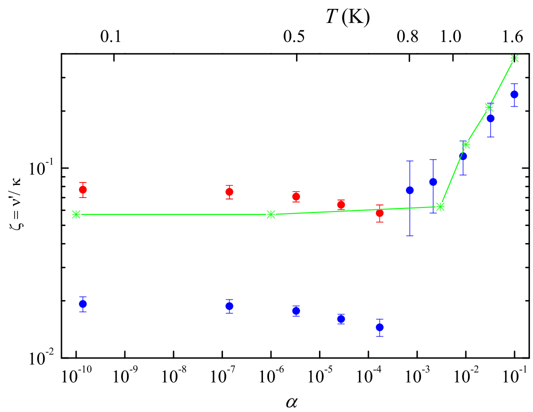

FIG. 1: Effective viscosity $\zeta = \nu'/\kappa$ vs. mutual friction parameter $\alpha(T)$ for grid turbulence (circles, see text). Green asterisks show the values of ζ for the decay of random tangles simulated numerically for different α [11] (the value $\zeta = 0.057$, plotted here at $\alpha = 10^{-10}$ and $\alpha = 10^{-6}$, was actually computed for $\alpha = 0$).

kept equal to Kolmogorov's value for HIT (it is determined by the Kolmogorov constant C).

In order to make quantitative comparison of the rates of decay obtained in different experiments, we applied this one-parameter approach, Eq. 4 to our data $L \propto t^{-3/2}$ from Fig. 4 of the main paper, with an appropriate choice of the value of the cut-off wavenumber k_1 that should depend on the BC. Namely, for no-slip BC, one would expect $\lambda = d$, i. e. $k_1 = 2\pi/d$ (as was used by [1] while for slip BC, $\lambda = 2d$, i. e. $k_1 = 2\pi/2d = \pi/d$.

In Fig. 1 we plot the resulting values of $\zeta(\alpha)$ for grid turbulence. The blue symbols show the values of ζ obtained with $k_1 = 2\pi/d$, while the red ones, for $T \leq 0.8$ K only, correspond to the values of ζ obtained using $k_1 = \pi/d$ (suitable for the slip BC at low temperatures). With the latter choice, the discontinuous drop in apparent data has vanished, and the dependence $\zeta(T)$ becomes smooth. Furthermore, it is close to the values $\zeta(0) = 0.08$ measured experimentally [10] and $\zeta(0) = 0.06$–0.10 calculated numerically [11, 12] (Fig. 1) for Vinen QT.

Unfortunately, several assumptions of the above approach are unrealistic. First of all, the description in terms of a unique k-spectrum while the system is strongly spatially-inhomogeneous (for instance, due to a particular shape of the container, or to the presence of a turbulent boundary layer) cannot be applied. But even if a one-for-all k-spectrum could be used, the Kolmogorov spectrum, Eq. 1, [6–8] was derived for wavenumbers k far away from both those for forcing and dissipation. Hence, it is not expected to apply near the cut-off wavenumber k_1, and there is no reason why the total energy and its flux should be described by Eq. 2 and Eq. 3. We thus conclude that Eq. 4 can only yield the prefactor A correct to the order of magnitude. Any numerical agreement must be purely accidental.

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